Modelling for stochastic processes with invariant functions

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My talk is devoted to the construction method for stochastic process with both strong random perturbations and invariant functions. Applying this approach to modelling, we can construct a model for the process under study based on their necessary the same properties property described in the form of an invariant function, and initial conditions only. This method also allows us to pass from a deterministic process under study to a controlled stochastic one, while preserving the constant values for given functions dependent on this process. Several examples illustrating the possibilities of the method will be presented. The proposed method for constructing a model is well adapted for use on a computer.

We use a system of diffusion Itô's equations with Poisson jumps

$$dX_i(t) = a_i\big(t, \mathbf{X}(t)\big)\, dt + b_{i,k}\big(t, \mathbf{X}(t)\big)\, d\mathrm{X}_k(t) + \int_{\mathbb{R}^m} g_i\big(t, \mathbf{X}(t), \gamma\big)\nu(dt, d\gamma),$$
$$\mathbf{X}(0) = \mathbf{X}_0, \qquad i = 1, ..., n, \quad n \geq 2, \tag{1}$$

where

$$a_i(t, \mathbf{x}) \in \mathcal{C}^{1,1}_{t,x}, \quad b_{ij}(t, \mathbf{x}) \in \mathcal{C}^{1,2}_{t,x}, \quad g_i(t, \mathbf{x}, \gamma) \in \mathcal{C}^{1,2,1}_{t,x,\gamma},$$

and $\mathbf{X}(t, \mathbf{X}_0, \omega) \in \mathbb{R}^n$ is a solution of Eq. (1), and

$$u\big(t, \mathbf{X}(t, \mathbf{X}_0, \omega), \omega\big) = u\big(0, \mathbf{x}_0\big), \qquad (\mathbf{P} - \text{a.}s.)$$

for every solution $X(t) = X(t, x_0, \omega)$ for Eq. (1). The function $u\big(t, \mathbf{X}(t), \omega\big)$ is an invariant one.

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